# **UPSC Mains Mathematics Optional Paper-I Syllabus**

## (1) Linear Algebra:

Vector spaces over R and C, linear dependence and independence, subspaces, bases, dimensions, Linear transformations, rank and nullity, matrix of a linear transformation.

Algebra of Matrices; Row and column reduction, Echelon form, congruence's and similarity; Rank of a matrix; Inverse of a matrix; Solution of system of linear equations; Eigenvalues and eigenvectors, characteristic polynomial, Cayley-Hamilton theorem, Symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices and their eigenvalues.

## (2) Calculus:

Real numbers, functions of a real variable, limits, continuity, differentiability, mean-value theorem, Taylor's theorem with remainders, indeterminate forms, maxima and minima, asymptotes; Curve tracing; Functions of two or three variables; Limits, continuity, partial derivatives, maxima and minima, Lagrange's method of multipliers, Jacobian.

Riemann's definition of definite integrals; Indefinite integrals; Infinite and improper integral; Double and triple integrals (evaluation techniques only); Areas, surface and volumes.



## (3) Analytic Geometry:

Cartesian and polar coordinates in three dimensions, second degree equations in three variables, reduction to Canonical forms; straight lines, shortest distance between two skew lines, Plane, sphere, cone, cylinder, paraboloid, ellipsoid, hyperboloid of one and two sheets and their properties.

## (4) Ordinary Differential Equations:

Formulation of differential equations; Equations of first order and first degree, integrating factor; Orthogonal trajectory; Equations of first order but not of first degree, Clairaut's equation, singular solution.

Second and higher order liner equations with constant coefficients, complementary function, particular integral and general solution.

Section order linear equations with variable coefficients, Euler-Cauchy equation; Determination of complete solution when one solution is known using method of variation of parameters.

Laplace and Inverse Laplace transforms and their properties, Laplace transforms of elementary functions. Application to initial value problems for 2nd order linear equations with constant coefficients.

## (5) Dynamics and Statics:

Rectilinear motion, simple harmonic motion, motion in a plane, projectiles; Constrained motion; Work and energy, conservation of energy; Kepler's laws, orbits under central forces.

Equilibrium of a system of particles; Work and potential energy, friction, Common catenary: Principle of virtual work; Stability of equilibrium, equilibrium of forces in three dimensions.

#### (6) Vector Analysis:

Scalar and vector fields, differentiation of vector field of a scalar variable; Gradient, divergence and curl in cartesian and cylindrical coordinates; Higher order derivatives; Vector identities and vector equation.

Application to geometry: Curves in space, curvature and torsion; Serret-Furenet's formulae.

Gauss and Stokes' theorems, Green's identities.

